Analysis 1, Summer 2024 List 0 Algebra review

1. Which of the following are true for all real values of the variables (or, at least, all real values for which both sides of the equation are defined)?

(a)
$$2x = x + x$$
 True
(b) $2(x + y) = 2x + y$ False
(c) $(x - y)^2 = x^2 - 2xy + y^2$ True
(d) $\frac{6+a}{2} = 3 + \frac{a}{2}$ True
(e) $-(y + 2) = -y + 2$ False
(f) $-(a + b)^2 = (-a + b)^2$ False
(g) $x^3 + 3x = x + x$ False
(h) $k^{-2} = \frac{1}{k^2}$ True
(i) $x^{a+2} = x^a \times x^2$ True
(j) $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$ False

(k) $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ True for non-negative numbers. If a < 0 or b < 0 or both, then this involves complex numbers, which are not part of this course.

(l)
$$\log(a+b) = \log(a) + \log(b)$$
 False

- (m) $\log(a \cdot b) = \log(a) + \log(b)$ True for positive numbers.
- (n) $\log(a \cdot b) = \log(a) \cdot \log(b)$ False
- 2. Compute the following values:

(a)
$$\cos(30^{\circ}) = \sqrt{3/2}$$

(b) $\cos(45^{\circ}) = 1/\sqrt{2} \text{ or } \sqrt{2/2}$

- (c) $\cos(60^\circ) = 1/2$
- (d) $\cos(\pi/3) = 1/2$ (same as previous)

(e)
$$\cos(\pi/2) = 0$$

(f)
$$\sin(8\pi/3) = \sqrt{3}/2$$

- (g) $\sqrt{10000} = 100$
- (h) $10000^{1/2}$ 100 (same as previous)
- (i) $\sqrt[4]{10000} = 10$

(j)
$$625^{-1/2} = 1/25$$

(k) $\log_7(49) = 2$

(ℓ) $\log_4(1024) = 5$ (m) $\log_9(3) = 1/2$ (n) $\log_k(1) = 0$ (o) $\ln(1) = 0$ (p) $\ln(\sqrt{e}) = 1/2$ (q) $\log_4(16^3) = 6$

3. Re-write $\frac{x^4}{\sqrt{x^{13}}}$ in the form x^{\Box} . $x^{-5/2}$ (That box is not a mistake in the file.)

4. Re-write each of the following in the form $\Box x^{\Box}$, or state that this is not possible. You can assume x > 0 if necessary.

(a)
$$x^3 \cdot x^7 = x^{10}$$

(b) $x^3x^7 = x^{10}$ (same as previous)
(c) $x^3 + x^7$ not possible
(d) $x^3 - x^7$ not possible
(e) $x^3/x^7 = x^{-4}$
(f) $\frac{x^3}{x^7} = x^{-4}$ (same as previous)
(g) $(x^3)^7 = x^{21}$
(h) $3x^5 + (x^5)^2$ not possible
(i) $3x^{10} + (x^5)^2 = 4x^{10}$
(j) $\sqrt{x^4} = x^2$
(k) $\sqrt{x^6 + x^4}$ not possible
(l) $\sqrt{x^6} + 8x^3 = 9x^3$
(m) $x^3 \cdot \sqrt[3]{x} = x^{10/3}$
(n) $\ln(e^x) = x$
(o) $\ln(e^{6x}) = 6x$
(p) $e^{9\ln(x)} = x^9$
(q) $e^{9\ln(x)+1} = ex^9$
(r) $e^{9\ln(x)} + 1$ not possible
(s) $\frac{\sqrt{4e^{\ln(9x)+10}}}{e^5} = 6x^{1/2}$

5. Re-write $\log_{10}(2^9)$ using the natural logarithm (ln). $\frac{\ln(512)}{\ln(10^6)}$

$$\frac{2}{0}$$
 or $\frac{9\ln(2)}{\ln(10)}$

- 6. Expand each of the following:
 - (a) $(a-b)^2 = a^3 2ab + b^2$
 - (b) $(1+x)^2 = 1+2x+x^2$
 - (c) $(2-x)^3 = 8 12x + 6x^2 x^3$
 - (d) $(6+h)^3 = 216 + 108h + 18h^2 + h^3$
- 7. Simplify $\frac{(6+h)^3 216}{h}$ if $h \neq 0$. $108 + 18h + h^2$
- 8. If a point on the line

$$y = -\frac{1}{3}(x-6) + 8$$

has an x-value of 15, what is its y-value? 5

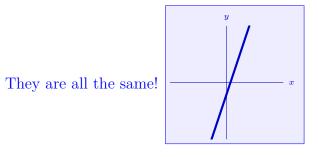
9. If a point on the line

$$y = -\frac{1}{3}(x - 6) + 8$$

has an x-value of 6, what is its y-value? 8

- 10. Graph each of the following:
 - (a) y = 3(x 1) + 2
 - (b) y = 3x 1
 - (c) y 2 = 3(x 1)
 - (d) y + 1 = 3x
 - (e) 3x y = 1

(f)
$$x = (y+1)/3$$



11. Give an example of a point that is on the line

$$y - 17 = 38(x - 12).$$

There are literally infinitely many correct answers. The easiest to find is (12, 17) because those numbers occur in the equation in such a way that

$$\underline{17} - 17 = 38(\underline{12} - 12).$$

12. Describe the shape of y = 7 in words. horizontal line Describe x = -2 in words. vertical line 13. Give an equation for the line through the point (-6, 5) with slope 2. y = 5 + 2(x+6) or y = 2x + 17

14. Give an equation for each of the following:

(a) the line through (1,3) with slope 5. y = 3 + 5(x - 1) or y = 5x - 2
(b) the line through (0, -9) with slope ²/₅. y = ²/₅x - 9
(c) the line through (-4.2, 6.1) with slope 8.88. y = 6.1 + 8.88(x + 4.2) or y = 8.88x + 43.396
(d) the line through (5,1) with slope -3. y = 1 - 3(x - 5) or y = -3x + 16
15. A line passes through both (4, 4) and (8, 2). What is its slope? -1/2

16. Give an equation for the line through (1,7) and (10, -6). $y = 7 - \frac{13}{9}(x-1) \text{ or } y = -6 - \frac{13}{9}(x-10) \text{ or } y = -\frac{1}{3}x + \frac{76}{9}$

17. For $f(x) = \begin{cases} x+1 & \text{if } x < 4 \\ x^2 & \text{if } x \ge 4 \end{cases}$, what is the value of f(4)? $4^2 = 16$

18. Draw a graph of the piecewise function $\left\{ \right.$

$$\begin{array}{ll} x & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x \leq 2 \\ 6-x & \text{if } x > 2 \end{array} .$$

